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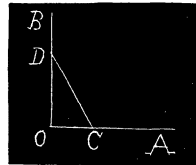
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Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let  $OA=a$ ,  $OB=b$ ,  $OD=u$ ,  $OC=x$ ,  $m$ =mass of particle at  $C$ ,  $n$ =mass of particle at  $D$ ;  $A, B$  the positions of the particles at the beginning of motion;  $C, D$  their positions at any time  $t$ ;  $v, v_1$  their velocities at  $C, D$ . Then the equations of motion are



$$\text{for } C, \frac{d^2x}{dt^2} = \frac{nx}{(x^2+u^2)^{\frac{3}{2}}} = F, \text{ since } s=x,$$

$$\text{for } D, \frac{d^2u}{dt^2} = \frac{mu}{(x^2+u^2)^{\frac{3}{2}}} = f, \text{ since } \sigma=u. \text{ But } v=Ft, v_1=ft.$$

$$\therefore v_1 = \frac{mu v}{nx}. \text{ Also } \frac{a-x}{v} = \frac{b-u}{v_1} = \frac{(b-u)nx}{mu v}.$$

$$\therefore u = \frac{bnx}{am-mx+nx} \dots (1). \quad \therefore \frac{d^2x}{dt^2} = \frac{n(am-mx+nx)^3}{x^2[a^2n^2+(am-mx+nx)^2]^{\frac{3}{2}}} \dots (2).$$

From (1),  $u=0$  when  $x=0$ . Therefore, the particles both arrive at  $O$  at the same time. Hence  $C$  moves over distance  $a$ , and  $D$  moves over distance  $b$  before they meet. The time is found by integrating (if possible) (2) twice.

$$\text{If } m=n, \frac{d^2x}{dt^2} = \frac{a^3m}{x^2 \sqrt{[(a^2+b^2)^3]}}.$$

$$\therefore \left(\frac{dx}{dt}\right)^2 = \frac{2a^3m}{(a^2+b^2)^{\frac{3}{2}}} \left(\frac{1}{x} - \frac{1}{a}\right) = \frac{2a^2m}{(a^2+b^2)^{\frac{3}{2}}} \cdot \frac{a-x}{x}.$$

$$\therefore t = \frac{(a^2+b^2)^{\frac{3}{2}}}{a\sqrt{2m}} \int_0^a \sqrt{\frac{x}{a-x}} dx = \frac{\pi(a^2+b^2)^{\frac{3}{2}}}{2\sqrt{(2m)}}.$$

$$\text{If } a=b \text{ and } m=\text{unity}, t=\pi(\frac{1}{2}a^2)^{\frac{3}{2}}.$$

#### AVERAGE AND PROBABILITY.

92. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pa.

A circular field, radius  $r$ , is divided into four *equal* parts, by concentric circles and three concentric rings. From the center of this field are fired *at random*, and with such a velocity as not to produce a range greater than the radius of the field,  $m=1000$  projectiles of the *same* kind. How many projectiles should have fallen into each one of these four equal parts of the field?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

The range  $= (v^2/g)\sin 2\theta$ , where  $\theta$ =angle of elevation. Greatest range  $= v^2/g = r$ .

$$\therefore v^2 = gr. \quad \therefore (v^2/g)\sin 2\theta = r\sin 2\theta.$$

The radii of the three concentric circles are  $\frac{1}{2}r\sqrt{3}$ ,  $\frac{1}{2}r$ ,  $\frac{1}{2}r$ , respectively.

$$\therefore r\sin 2\theta = \frac{1}{2}rn, \text{ suppose. Solving, we get } \sin \theta = \frac{1}{2}\sqrt{[2 \pm \sqrt{4-n^2}]}$$

$$\text{When } n = \sqrt{3}, \sin \theta = \frac{1}{2}\sqrt{(2 \pm 1)}. \quad \therefore \theta = \frac{1}{3}\pi \text{ or } \frac{2}{3}\pi.$$

$$\text{When } n = \sqrt{2}, \sin \theta = \frac{1}{2}\sqrt{(2 \pm \sqrt{2})}. \quad \therefore \theta = 3\pi/8 \text{ or } \frac{5}{8}\pi.$$

$$\text{When } n = 1, \sin \theta = \frac{1}{2}\sqrt{(2 \pm \sqrt{3})}. \quad \therefore \theta = 5\pi/12 \text{ or } \frac{7}{12}\pi.$$

Chance that all fall into outer ring =  $p$ .

$$\therefore p = \frac{r \int_{\frac{1}{3}\pi}^{\frac{2}{3}\pi} \sin 2\theta d\theta}{r \int_0^{\frac{1}{2}\pi} \sin 2\theta d\theta} = \frac{1}{2}. \quad p_1 = \frac{r \int_{\frac{1}{3}\pi}^{3\pi/8} \sin 2\theta d\theta}{r \int_0^{\frac{1}{2}\pi} \sin 2\theta d\theta} = \frac{1}{2}\sqrt{2},$$

the chance that all will fall in the two outer rings.

$\therefore P = p_1 - p = \frac{1}{2}(\sqrt{2} - 1)$  = the chance that all will fall in the second ring from without.

$$p_2 = \frac{r \int_{\frac{1}{2}\pi}^{5\pi/12} \sin 2\theta d\theta}{r \int_0^{\frac{1}{2}\pi} \sin 2\theta d\theta} = \frac{1}{2}\sqrt{3}, \text{ the chance that all will fall in three outer rings.}$$

$\therefore P_1 = p_2 - p_1 = \frac{1}{2}(\sqrt{3} - \sqrt{2})$  = chance that all fall in third ring from without.

$\therefore P_2 = 1 - p_2 = \frac{1}{2}(2 - \sqrt{3})$  = chance that all fall in small circle around the center.

$\therefore$  Number to fall in each space is proportional to these chances, or as  $1:(\sqrt{2}-1):(\sqrt{3}-\sqrt{2}):(2-\sqrt{3})$ .

$\therefore$  Number in outer ring =  $\frac{1}{2}m = 500$ .

Number in second ring =  $m(\sqrt{2}-1)/2 = 500(\sqrt{2}-1) = 207.1065$ .

Number in third ring =  $m(\sqrt{3}-\sqrt{2})/2 = 500(\sqrt{3}-\sqrt{2}) = 158.9185$ .

Number in inside circle =  $m(2-\sqrt{3})/2 = 500(2-\sqrt{3}) = 133.9750$ .

93. Proposed by LON C. WALKER, Assistant in Mathematics, Leland Stanford, Jr. University, Palo Alto, Cal.

In Problem 75, required the average area of the circle inscribed in the triangle.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let  $r$  = radius of inscribed circle.

$$\text{Then } \frac{1}{2}pr = \text{area} = \frac{1}{2}r[x+y+\sqrt{(x^2+y^2)}] = \frac{1}{2}xy.$$

$$\text{But } x = \frac{p^2-2py}{2(p-y)}. \quad \therefore \frac{1}{2}pr = \frac{p(p-2y)y}{4(p-y)}.$$

$$\therefore r = \frac{(p-2y)y}{2(p-y)}, \quad \pi r^2 = \frac{\pi y^2(p-2y)^2}{4(p-y)^2}.$$